V. MULTIPLICATION OF LARGE NUMBERS.

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In a recent number of the MONTHLY (*1921*, 114) I gave computations for $e^{\pm\pi}$, $e^{\pm\pi/2}$, $e^{\pm\pi/3}$, $e^{\pm\pi/6}$ to over fifty places of decimals. After mailing the article I set to work to form the product $e^{-\pi}e^{\pi}$, which should approximate to unity. The computation was performed in a single afternoon and evening and resulted in 1 followed by 52 zeros before the appearance of significant figures 3859···
It seems certain, therefore, that the earlier computations are correct to more than 50 decimal places,—my original tacit goal.

It may be of interest to give an account of the scheme used for multiplying two large numbers in such a manner as to avoid errors.

Buy suitable coördinate paper. Cut up long strips of cardboard about half an inch wide. Lay a strip of cardboard on the coördinate paper so as to write the digits properly spaced. Multiply the multiplicand by one of the nine digits 1, 2, ···, 9, one such result to be written on one strip of cardboard; so that nine strips will contain all the partial products to be used. No error can remain on these strips after careful inter-comparison and mutual checking, e.g., $6 = 2 \times 3 = 5 + 1$, etc. Then place a properly spaced copy of the multiplier above and to the right of the large sheet of coördinate paper, and fasten it in position.

Commence with the extreme left-hand digit of the multiplier and select the corresponding strip of cardboard. Place the extreme right-hand digit of the cardboard exactly below or under the left-hand digit of the multiplier and copy on the coördinate sheet the figures on the cardboard. There is no mental work; and the eye cannot miss a single mistake, since each digit copied on the co-ordinate sheet is directly under the same digit on the cardboard.

Now place the next strip indicated by the multiplier one space lower and one space further to the right, and copy as before. Continue this process for successive figures of the multiplier, omitting unnecessary figures falling beyond the right side of the coördinate sheet.

No error should arise in adding the columns on the coördinate sheet because of repetition and other obvious means of checking. As a matter of fact, I have, as yet, made not a single mistake that I did not detect before proceeding further.

The illustration given shows the actual copy of the coördinate sheet as used for the multiplication of the numbers

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[1] The text of this article was adapted by the editor from a private communication of Professor Uhler.—EDITOR.

$e^\pi = $ 23.14069 26327 79269 00572 90863 67948 54738 02661 06242 60021 16
$e^{-\pi} = $ 0.04321 39182 63772 24977 44177 37171 72801 12757 28109 81063 30854

8642	78365	27544	49954	88354	74343	45602	25514	56219	62126	61707 00256 2
1296	41754	79131	67493	23253	21151	51840	33827	18432	94318	99256 05038 43
43	21391	82637	72249	77441	77371	71728	01127	57281	09810	63308 53501 281
17	28556	73055	08899	90976	70948	68691	20451	02912	43924	25323 41400 5124
	25928	35095	82633	49864	65064	23030	36806	76543	68658	86379 85121 00769 −
	3889	25264	37395	02479	69759	63454	55521	01481	55298	82956 97768 15115 +
	86	42783	65275	44499	54883	54743	43456	02255	14562	19621 26617 07003 −
	25	92835	09582	63349	86465	06423	03036	80676	54368	65886 37985 12101 −
	1	29641	75479	13167·	49323	25321	15151	84033	82718	43294 31899 25605
		8642	78365	27544	49954	88354	74343	45602	25514	56219 62126 61707
		3024	97427	84640	57484	20924	16020	20960	78930	09676 86744 31597 +
		302	49742	78464	05748	42092	41602	02096	07893	00967 68674 43160 −
		38	89252	64373	95024	79697	59634	54555	21014	81552 98829 56978 −
			86427	83652	75444	99548	83547	43434	56022	55145 62196 21266 +
			25928	35095	82633	49864	65064	23030	36806	76543 68658 86380 −
			3889	25264	37395	02479	69759	63454	55521	01481 55298 82957 −
			2	16069	59131	88612	48872	08868	58586	40056 37864 05491 −
				30249	74278	46405	74842	09241	60202	09607 89300 96769 −
				864	27836	52754	44995	48835	47434	34560 22551 45622 −
				388	92526	43739	50247	96975	96345	45552 10148 15530 −
				3	45711	34611	01779	98195	34189	73738 24090 20582 +
					25928	35095	82633	49864	65064	23030 36806 76544 −
					1296	41754	79131	67493	23253	21151 51840 33827 +
					259	28350	95826	33498	64650	64230 30368 06765 +
					30	24974	27846	40574	84209	24160 20209 60789 +
					3	88925	26437	39502	47969	75963 45455 52101 +
						17285	56730	55088	99909	76709 48686 91205 −
						3457	11346	11017	79981	95341 89737 38241 −
						216	06959	13188	61248	87208 86858 58640
						17	28556	73055	08899	90976 70948 68691 +
						3	02497	42784	64057	48420 92416 02021 −
							12964	17547	91316	74932 32532 11515 +
							3457	11346	11017	79981 95341 89737 +
							8	64278	36527	54449 95488 35474 +
							2	59283	50958	26334 98646 50642 +
								25928	35095	82633 49864 65064 +
								432	13918	26377 22497 74418 −
								25	92835	09582 63349 86465 −
									86427	83652 75444 99549 −
									17285	56730 55088 99910 −
									864	27836 52754 44995 +
									259	28350 95826 33499 −
										8642 78365 27544 +
										432 13918 26377 +
										43 21391 82638 −
										25 92835 09583 −
										43213 91826 +
										30249 74278 +
										2592 83510 −
										388 92526 +
										30 24974 +
										2 16070 −

1.0000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00038 59222 80390